$\boxed{\text{Solutions Chapter 2 Exam MAC 2233}}$

① $f(x) = 2x^2 + 8x + 5$

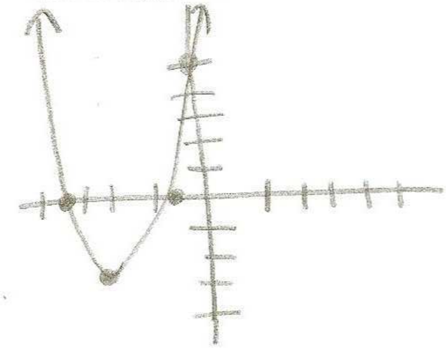
$a = 2 \Rightarrow$ upward parabola

$\Rightarrow$ minimum (at vertex)

$h = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$

$K = 2(-2)^2 + 8(-2) + 5$
$= 2(4) - 16 + 5$
$= -3$

$\boxed{(-2, -3) = D}$



② using equation from #1

vertex $= (-2, -3)$

$\boxed{\text{x-int}}$ $y = 0$

$0 = 2x^2 + 8x + 5$

solve the quadratic equation (method of choice)

$x = h \pm \sqrt{\frac{-k}{a}} = -2 \pm \sqrt{\frac{-(-3)}{2}} = -2 \pm \sqrt{\frac{3}{2}} \cong -0.78$ & -3.22

$\boxed{\text{y-int}}$ $x = 0$ $(0, 5)$

$f(0) = 2(0)^2 + 8(0) + 5$
$= 2(0) + 8(0) + 5 = 5$

③ demand $p^2 + q = 104 \Rightarrow q = 104 - p^2$

supply $2p - q = 16 \Rightarrow -q = 16 - 2p$ so $q = 2p - 16$

ⓐ Equilibrium

supply = demand

$p = 10$

or $\cancel{p = -12}$

$\boxed{\$10 \text{ per unit}}$

$104 - p^2 = 2p - 16$
$0 = p^2 + 2p - 120$
$0 = (p - 10)(p + 12)$

Ⓑ

Quantity $= q$

$q = 104 - (10)^2 = 4$

so $\boxed{400 \text{ units}}$

(4) check each equation to see if (3,5) & (4,10)

a) $y=2(5)^{x-3}$

x	y
3	2
4	10

b) $y= 5(2)^{x-3}$

x	y
3	5
4	10

c) $y=5(2)^{x/3}$

x	y
3	10
4	12.6

d) $y=2(5)^{x+3}$

x	y
3	31250
4	156250

(5) \$4000 at 4.8% compounded quarterly (n=4)
find value in 10 years.

C

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = (4000)\left(1+\frac{0.048}{4}\right)^{4(10)}$

$A \cong (4000)(1.012)^{40} \cong \boxed{\$6445.85}$

(6) $R(t) = 13.62e^{.15t}$

D

$t=0$ 2000
$t=5$ 2005

$R(5) = 13.62e^{.15(5)} \cong 28.8$

(7) \$20,000 in 5 yrs
5% compounded continuously

$A = Pe^{rt}$

C

$20{,}000 = Pe^{(.05)(5)}$

$20{,}000 = P(1.284)$

\$15,576.32 $= P$

(8) 2000 mosquitos initially
2hrs later → 300 mosquitos
Find 5 hrs later

exponential decay model

$\boxed{P(t) = 2000e^{-0.9486t}}$

$P = P_0e^{kt}$ with $(0,2000)$ $(2,300)$

$P(5) = 2000e^{-0.9486(5)}$
$\cong 17.42$ mosquitos

$P = 2000e^{kt}$

$300 = 2000e^{k(2)}$

$\frac{300}{2000} = e^{2k}$

$0.15 = e^{2k}$

$\ln 0.15 = \ln e^{2k}$

$\ln 0.15 = 2k \cancel{\ln e}$ 1

$\frac{\ln 0.15}{2} = k$

$k = -0.9486$

so

C

(9) \$16,500 in beginning of 2000
\$12,375 end of 2000
\$9,281 end of 2001

create an exponential model

predict end of 2004

let t = time in months

end of 2000	12
2001	24
2002	
2003	
2004	60

$P(t) = P_0 e^{kt}$

$P(t) = 16{,}500 e^{kt}$

$P(12) = 16{,}500 e^{k(12)} = 12{,}375$

$e^{12k} = 0.75$

$\ln e^{12k} = \ln 0.75$

$12k(\ln e) = \ln(0.75)$

$k = -0.02397$

$P(t) = 16{,}500 e^{-0.02397}$

check $P(24) = 16{,}500 e^{-0.02397} \approx 9282$ so ok

↑ end of 2001

$P(60) = 16{,}500 e^{-0.02394(60)}$

$\approx \boxed{\$3916.35}$

(10) recall $\log_b a = x \Leftrightarrow a = b^x$

so $32^{1/5} = 2$

$\boxed{\log_{32} 2 = \frac{1}{5}}$ (C)

(11) check each one

a) $\ln \frac{1}{e^8} = e^{-8}$

$\ln e^{-8} =$

$-8 \ln e =$

$-8 \neq e^{-8}$

so a fails

(A)

(12) $\log_5 (2x-3) = 2$

$5^2 = (2x-3)$

$25 = 2x - 3$

$28 = 2x$

$14 = x$

(13) $\frac{1}{2}$ life t in days

400 grams initial

290 grams in one week

$P = P_0 e^{kt}$

$290 = 400e^{k(7)}$

$\frac{290}{400} = e^{7k}$

$0.725 = e^{7k}$

$\ln 0.725 = \ln e^{7k}$

$\ln 0.725 = 7k \cancel{\ln e}^{1}$

$-0.04594 = k$

$\boxed{P = 400e^{-0.04594t}}$

$\frac{1}{2}$ life = when $\frac{1}{2}$ decays

$200 = 400e^{-0.04594t}$

$0.5 = e^{-0.04594t}$

$\ln 0.5 = \ln e^{-0.04594t}$

$\ln 0.5 = -0.04594t \cancel{\ln e}^{1}$

$15.088 = t$

so 15 days

(14) $T = 22 + 98(0.30)^{.2t}$

How long to cool to 50°C?

$50 = 22 + 98(0.30)^{.2t}$

$28 = 98(0.30)^{.2t}$

$\frac{28}{98} = (0.30)^{.2t}$

$0.2857 = (0.30)^{.2t}$

$\ln(0.2857) = \ln(0.30)^{.2t}$

$\ln(0.2857) = 0.2t \ln(0.30)$

$\frac{\ln(0.2857)}{\ln(0.30)} = 0.2t$

$1.041 = 0.2t$

$\boxed{5.2 = t}$

$\boxed{\text{minutes}}$

(15) $Q(t) = \dfrac{1000}{1 + 200e^{-kt}}$

50 students on 6th day (t=6)

(find k)

$50 = \dfrac{1000}{1 + 200e^{-k(6)}}$

$1 + 200e^{-6k} = \frac{1000}{50} = 20$

$1 + 200e^{-6k} = 20$

$200e^{-6k} = 20$

$e^{-6k} = 0.1$

$\ln e^{-6k} = \ln(0.1)$

$-6k = \ln(0.1)$

$k = +0.3838$

A

the key is incorrect

(16) $N(t) = \dfrac{500}{1+49e^{-0.21t}}$

(A) at beginning $(t=0)$

$$N(0) = \frac{500}{1+49e^{-0.21(0)}} = \frac{500}{1+49(1)} = \frac{500}{50} = \boxed{10}$$

(B) max = limiting factor = $\boxed{500}$

(C) on day 10

$$N(10) = \frac{500}{1+49e^{-0.21(10)}} = 71.4$$

so $\boxed{71^{st} \text{ day}}$

(17) When will the population be 300?

$$300 = \frac{500}{1+49e^{-0.21t}}$$

$$1+49e^{-0.21t} = \frac{500}{300} = \frac{5}{3}$$

$-1 \qquad\qquad -1$

$$\frac{1}{49} \cdot 49e^{-0.21t} = \frac{2}{3} \cdot \frac{1}{49}$$

$$e^{-0.21t} = \frac{2}{147}$$

$$\ln e^{-0.21t} = \ln\left(\frac{2}{147}\right)$$

$$-0.21t \cancel{\ln e}^{\,1} = \ln\left(\frac{2}{147}\right)$$

$$-0.21t = -4.297$$

$$t = \frac{-4.297}{-0.21}$$

$$\boxed{t = 20.46 \text{ days}}$$

BONUS

$$m = \log \frac{I}{I_0}$$

$$6 = \log_{10} \frac{I_6}{I_0} \quad \& \quad 3 = \log_{10} \frac{I_3}{I_0}$$

using the def. of log

$$10^3 = \frac{I_3}{I_0} \quad \& \quad 10^6 = \frac{I_6}{I_0}$$

$$I_0 \cdot 10^3 = I_3 \qquad I_0 \cdot 10^6 = I_6$$

ratio comparison

$$\frac{I_6}{I_3} = \frac{I_0\, 10^6}{I_0\, 10^3}$$

$$\frac{I_6}{I_3} = \frac{10^6}{10^3} = \boxed{1000}$$